Statistics 5314: Homework 1

For each homework assignment, turn in at the beginning of class on the indicated due date. Late assignments will only be accepted with special permission. Write each problem up *very* neatly using LaTeX.

Problem 1

Recall that a linear congruential number generator has the form:

$$x_n = (ax_{n-1} + c) mod(m),$$

where

- $0 \leq a \leq m$: is the multiplier,
- $0 \leq c \leq m$: is the increment,
- $0 \leq m$: is the modulus,
- x_0: is the seed.

Part a

Implement this number generator.

Part b

For three (3) various value sets of a, c, m and x_0 plot *sequential triples*. At least two of your experiments need to show clear nonuniform patterns.

Problem 2

Plot sequential triples using the Mersenne Twister algorithm. In Matlab, this is the default generator. A C/C++ implementation can be found at (http://www.bedaus.net/mtrand/). You may use any software package that you're comfortable with.

Problem 3

For this exercise, you will implement a series of Monte Carlo estimates, each using $N = \{10, 100, 10000\}$ samples. Additionally, you will report the Monte Carlo variance associated with each of your estimates.

Part a

Approximate:

$$\int_0^{\pi} e^{sin(x)} 5e^{-5x} dx.$$

Part b

Approximate:

$$Pr(1.7 \leq X \leq 5.2),$$

where $X \sim Normal(\mu = 1, \sigma = 2)$.

Problem 3

Consider the function:

$$\int_0^1 4\sqrt{1 - x^2} dx.$$

What is the minimum Monte Carlo sample size for which the 5th decimal value is reliably a 9? Note: you are estimating $\pi \approx 3.14159...$ here.

Problem 4

Consider two circles, centered at $(0, 2)$ and $(0, -1)$, with radii $(r_1 = 2, r_2 = 3)$, respectively. Using Monte Carlo, approximate the area which is common to both circles.