Statistics 5444: Homework 0

For each homework assignment, turn in at the beginning of class on the indicated due date. Late assignments will only be accepted with special permission. Write each problem up *very* neatly (LaTeX is preferred). Show all of your work.

Problem 1

Read *When Did Bayesian Inference Become "Bayesian"?* (See webpage).

Part a

Who originally coined the term Bayesian?

Part b

Who was the founder of Bayesian statistics? Justify why you think so.

Problem 2

Consider observing data X. Assume you have a parametric model (i.e. sampling function, density/mass function, probability model, etcÉ) which depends on the parameter θ. The likelihood function (Recall that $p(X|\theta) \propto L(\theta|X)$) is defined over the values of $\theta \in \{-3, -2, -1, 0, 1, 2, 3, 4\}$. Likelihood and prior distribution values are given in Table **??**.

θ	-3	-2	-1	0	1	2	3	4
$L(\theta\|X)$:	0.5	2	1	3	1	3	2	0.5
$p(\theta)$:	0.1	0.3	0.05	0.15	0.05	0.1	0.2	0.05

Table 1: Likelihood and prior values

Part a

Compute the posterior distribution over the valid range of θ and plot the likelihood, prior and posterior in a single graph.

Part b

Compute $E[\theta|X]$.

Problem 3

Consider the function ("Likelihood Function"): $L(\mu|\sigma^2, \{x_1, \ldots, x_N\}) = \prod_{i=1}^{N} \frac{1}{\sqrt{(2\pi)}\sigma} e^{-\frac{1}{2}(x_i-\mu)^2/\sigma^2}$.

Part a

What value of μ maximizes $L(\mu|\sigma^2, \{x_1, \ldots, x_N\})$?

Part b

The 'kernel' of the Likelihood function is: $L(\mu|\sigma^2, \{x_1, \ldots, x_N\}) \propto \prod_{i=1}^N e^{-\frac{1}{2}(x_i-\mu)^2/\sigma^2}$. Rewrite this function using $\bar{x} = \sum_i^N x_i/N$ as opposed to using the product function and the full dataset: $\{x_1, \ldots, x_N\}$. It looks like: $e^{-\frac{1}{2}(\mu-c)^2/s}$ (you need to find c and s).

Problem 4

Bivariate Random Variables: Let x_1 and x_2 have a bivariate Normal distribution, which can be expressed by:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right),$$

where $\Sigma_{21} = \Sigma_{12}^T$. Note: I'm being really sloppy about the dimensionality of $x_1, x_2, \mu_1, \mu_2, \Sigma_{11}, \Sigma_{12}, \Sigma_{21}$, and Σ_{22}.

Part a

Label the dimensions and make sure everything is "conformable" (look that word up if you don't know it:)).

Part b

The Covariance Matrix $\mathbf{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ is Positive Definite.
What does it mean to be Positive Definite? Why is it important that $\mathbf{\Sigma}$ is Positive Definite?

Letting $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and $\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, **The Joint Normal Density Function** is:

$$f(x_1, x_2) = \frac{1}{\sqrt{(2\pi)^D|\mathbf{\Sigma}|}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathbf{T}}\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}.$$

Part c

What is the value for D in the term $\sqrt{(2\pi)^D|\mathbf{\Sigma}|}$?

Part d

$f(x_1)$ and $f(x_2)$ can each be shown to be Normals. What are the respective means and (co)variances for each of these? You may simply state the answers.

Part e

What is the dimension of $(\mathbf{x}-\boldsymbol{\mu})^{\mathbf{T}}\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$?